

# A study on the bending stress of the hollow sun gear in a planetary gear train<sup>†</sup>

Kyung-Eun Ko\*, Do-Hyeong Lim, Pan-Young Kim and Jinsoo Park

*Machinery Design Research Department, Hyundai Heavy Industries Co., LTD, Ulsan, 682-792, Korea*

(Manuscript Received May 2, 2009; Revised September 21, 2009; Accepted October 16, 2009)

## Abstract

Generally, planetary gear type traveling reduction gear is composed of multiple planetary gear stages and has a hollow sun gear in the last stage planetary gear. In designing reduction gear, it is important to evaluate accurately the bending stress at the tooth root of the sun gear as the sun gear is the weakest component in the reduction gear system. Although bending stress can be calculated easily using gear standard codes such as the American Gear Manufacturers Association (AGMA) and International Organization for Standardization (ISO) 6336 for almost all gears, meticulous calculation is needed for the hollow sun gear because of its low backup ratio (rim thickness divided by tooth height) and comparatively large root fillet radius. In this study, a finite element analysis (FEA) is carried out to investigate the effect of rim thickness and root fillet radius on bending stress at the tooth root of the hollow sun gear. In standard codes, bending stress at the tooth root is calculated linearly with a constant slope for the backup ratio below 1.2. However, the effect of the rim thickness on bending stress is more complex in the planetary gear system. Bending stresses calculated by FEA with various backup ratios and root filler radii are compared with the bending stresses calculated by the standard codes.

*Keywords:* AGMA; Backup ratio; Bending stress; Fillet radius; Hollow sun gear; ISO; Rim thickness

## 1. Introduction

Planetary gear trains are used widely in the machinery industry, especially in automotive and aerospace applications, because of its advantages such as compactness, coaxial design, and high performance.

The crawler excavator is equipped with a traveling reduction gear composed of multiple planetary gear stages. In the last planetary gear stage, the traveling reduction gear has a hollow sun gear, which is usually the weakest component in the system for tooth bending stress because of its intrinsic low backup ratio (rim thickness divided by tooth height).

Bending stress almost linearly increases as the backup ratio decreases. In this study, the actual effect of the backup ratio on the bending stress of the sun gear is investigated by direct structural analysis of a full reduction gear system for a crawler-type excavator.

The bending stress is affected by the root fillet radius as well. Thus, bending stresses are calculated for the various backup ratios and root fillet radii, and then compared with those calculated by the standard codes.

## 2. Bending stress calculation

### 2.1 Standard codes

The most common methods of gear design and analysis are based on international gear standards such as the American Gear Manufacturers Association (AGMA) and International Organization for Standardization (ISO), where the formulas for gear tooth bending stress calculations are included.

For example, for ISO 6336-3 [1], bending stress and nominal bending stress are calculated by Eq. (1) and Eq. (2), in which the effect of the backup ratio is considered by the rim thickness factor of  $Y_B$ , and the effect of root fillet radius is considered by the form factor of  $Y_F$  and the stress correction factor of  $Y_S$ . In the standards, these factors are calculated independently.

$$\sigma_F = \sigma_{F0} K_A K_V K_{F\beta} K_{F\alpha} \quad (1)$$

$$\sigma_{F0} = \frac{F_t}{b m_n} Y_F Y_S Y_{\beta} Y_B Y_{DT} \quad (2)$$

As shown in Fig. 1, the rim thickness factor is treated as constant 1.0 for the backup ratio above 1.2, and it almost linearly increases for the backup ratio below 1.2 in both AGMA and ISO standards.

<sup>†</sup> This paper was presented at the ICMĐT 2009, Jeju, Korea, June 2009. This paper was recommended for publication in revised form by Guest Editors Sung-Lim Ko, Keiichi Watanuki.

\*Corresponding author. Tel.: +82 52 202 0832, Fax.: +82 52 202 5495

E-mail address: kekopro78@hhi.co.kr

© KSME & Springer 2010

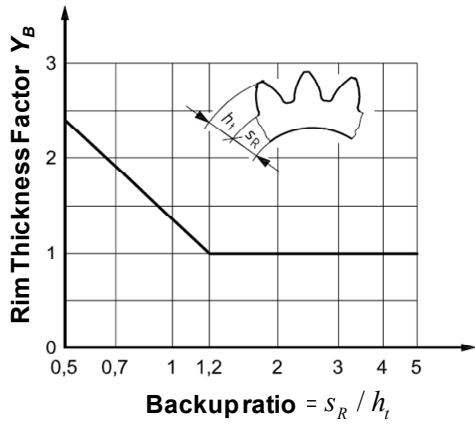


Fig. 1. The rim thickness factor versus backup ratio in the standard codes.

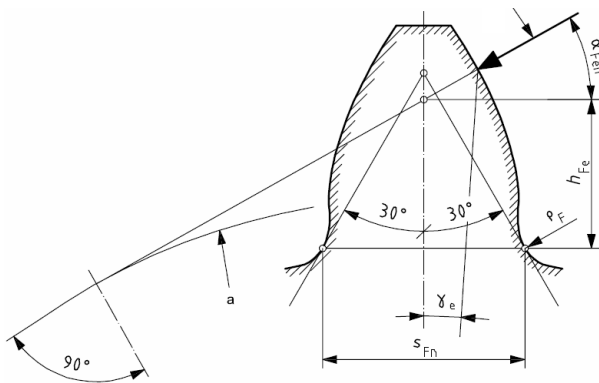


Fig. 2. Geometry parameters for the shape of the tooth root in the standard codes.

Meanwhile,  $Y_F$  and  $Y_S$  are calculated from many geometry parameters related to the root fillet radius according to Eq. (3) and Eq. (4). The parameters are shown in Fig. 2.

$$Y_F = \frac{6h_{Fe} \cos \alpha_{Fen}}{\left(\frac{s_{Fn}}{m_n}\right)^2 \cos \alpha_n} \quad (3)$$

$$Y_S = \left(1.2 + 0.13 \frac{s_{Fn}}{h_{Fe}}\right) \frac{s_{Fn}}{2\rho_F} \left[ \frac{1}{1.21 + \frac{2.3h_{Fe}}{s_{Fn}}} \right] \quad (4)$$

**2.2 FEA for planetary gear train**

To investigate precisely the influence of rim thickness and root fillet shape in hollow sun gear design, an integrated finite element analysis (FEA) including gear contact, roller bearing, carriers, housing, and shafts was carried out.

As there is always a contact point between two meshed gear surfaces, conventional FE analysis becomes problematic when applied to gear analysis. In this study, detailed FE analysis of the gear system was carried out using the three-dimensional multi-body contact analysis program, Calyx. Fig. 3 shows the

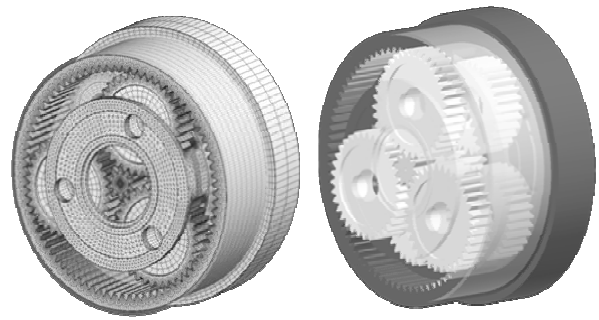


Fig. 3. FE model of power split-type of planetary gear set for a traveling device.

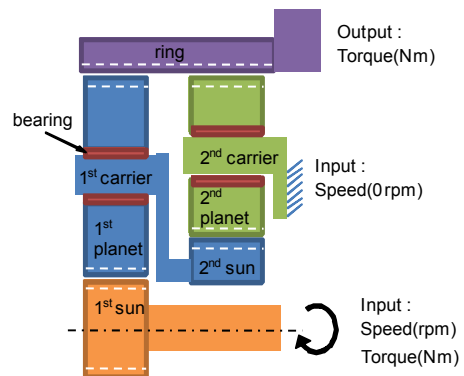


Fig. 4. Schematic of the planetary gear configuration and boundary conditions.

analysis model. This program uses an efficient contact solving algorithm by making a unique stiffness model, which is a combination of finite elements and contact theory [2, 3].

Fig. 4 shows how the analysis model of the two-stage power split-type planetary gear train for a traveling motor is prepared. This reduction gear is operated by a hydraulic motor, and the output torque of the hydraulic motor is exerted on the first sun gear. This type of gear has two power paths to maximize torque capacity: the first sun-first planet ring and the first sun-first planet-first carrier-second sun-second planet ring. The ring gear is connected to the travel device, and the second carrier is fixed to the motor housing; thus, the second planet rotates on its own axis only. Static analysis was carried out during one mesh cycle of the second sun gear, and all the stress levels of the different cases were compared at the same time of maximum stress (usually, this is the highest stress point of a single tooth contact point).

To obtain the results of the analysis, maximum principle stresses were searched at every calculation case near the tooth root fillet area to compare with the results calculated by the standard codes.

**3. Results and discussion**

The maximum principle stress distribution varies during the mesh cycles. Thus, the bending stress of the hollow sun gear from FEA is defined as the highest maximum principle stress

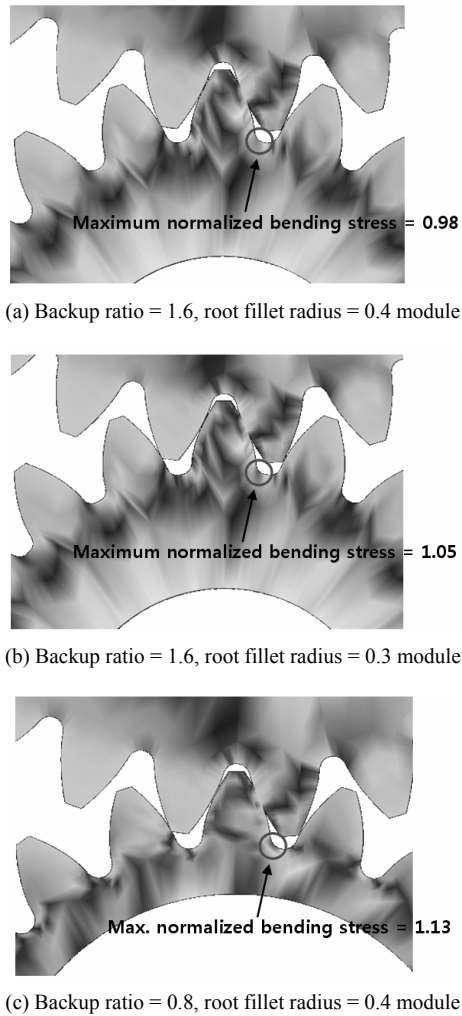


Fig. 5. Maximum principle stress distributions in the hollow sun gear.

in one mesh cycle. The bending stress occurred at the tooth root fillet as expected. Some snapshots of the highest maximum principle stress occurrence are shown in Fig. 5, where the maximum normalized bending stress is the maximum bending stress divided by the maximum stress for a backup ratio of 1.32 and fillet radius of 0.4 module.

Bending stress calculations by FEA have been attempted in some studies [4], but only two or three gear teeth have been modeled. In this study, all the structural effects of other mechanical components and real contact conditions were considered.

### 3.1 Effect of rim thickness

To compare the effect of backup ratio, all calculated maximum bending stresses are normalized by the maximum stress for a backup ratio of 1.2 and plotted to a backup ratio in Fig. 6.

It seems that the effect of the backup ratio above 1.2 may be negligible as indicated in the standards. However, the effect of the backup ratio below 1.2 may be considerably overestimated.

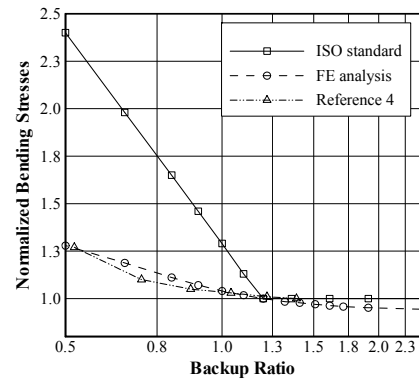


Fig. 6. Bending stress versus backup ratio (root fillet radius = 0.4 module).

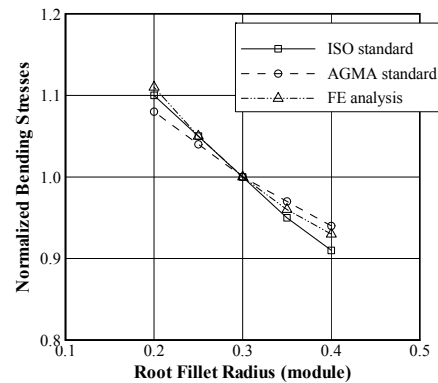


Fig. 7. Bending stress versus root fillet radius (backup ratio = 1.32).

For the range below 0.5, the standards do not give the guidelines. Meanwhile, it is noted in Ref. 5 that sudden catastrophic failure due to a crack through the rim thickness is prone to occur in case of a backup ratio below 0.5. Thus, the current standards seem to be conservative, as the backup ratio becomes smaller although the bending stress is not a direct cause of the rim through crack.

### 3.2 Effect of root fillet radius

The influence of the root fillet radius on the bending stresses is summarized in Fig. 7. This figure plots the max. bending stresses normalized by the max. stress for a fillet radius of 0.3 module as a function of root fillet radius.

Bending stress increases as the root fillet radius decreases, and the effects of root fillet radius calculated by FEA are similar to those obtained by the formulas of the standards. Generally, 0.2–0.3 modules are recommended, and this study investigated comparatively larger root fillet radius up to 0.4, which results in notable stress reduction.

### 3.3 Interaction between the effects of rim thickness and root fillet radius

To examine the interaction between the effects of rim thickness and root fillet radius on the bending stresses, the

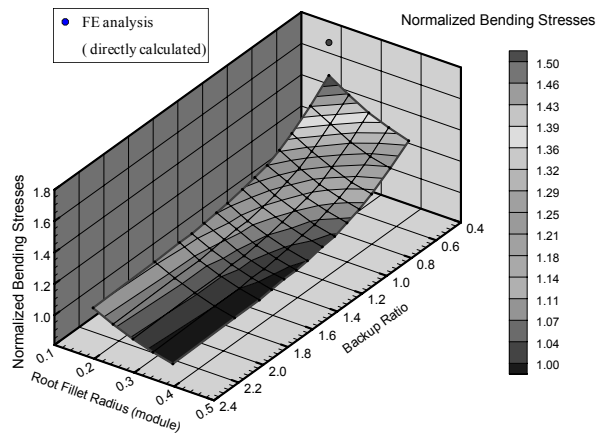


Fig. 8. Comparison between the direct calculation and estimation for bending stress.

bending stress in the case of the 0.5 backup ratio and 0.2 module fillet radius was directly calculated by FEA and estimated by extrapolating with the FEA results in Figs. 6 and 7.

The directly calculated value is plotted as a circle point in Fig. 8, which shows that the interaction of rim thickness and root fillet radius in a very low backup ratio and small fillet radius may increase the bending stress at the tooth root. It may also indicate that the standard codes do not properly include the interaction effects on bending stress.

Further study is needed to extract a more precise and general pattern for bending stress considering the interaction of those factors.

#### 4. Conclusions

In this study, the influence of rim thickness and root fillet radius on bending stress was investigated for the hollow sun gear in a planetary reduction gear system. The bending stress, the highest maximum principle stress in one cycle mesh, was calculated with a variation of backup ratio and root fillet radius by FEA to compare with standards such as ISO and AGMA. It was found that the consideration of backup ratio and root fillet radius in the standards was reasonable for the concerned hollow sun gear in the range of a backup ratio above 1.2. However, the rim thickness effect was overestimated in the range of a backup ratio between 0.5 and 1.2. The bending stress could become larger through the interaction of rim thickness and root fillet radius in a very low backup ratio and small fillet radius.

#### Nomenclature

$\alpha_{Fen}$  : Load direction angle

$\alpha_n$  : Normal pressure angle  
 $\rho_F$  : Radius of root fillet  
 $\sigma_F$  : Bending stress  
 $\sigma_{F0}$  : Nominal bending stress  
 $b$  : Face width  
 $F_t$  : Nominal tangential load  
 $h_{Fe}$  : Bending moment arm  
 $K_A$  : Application factor  
 $K_V$  : Internal dynamic factor  
 $K_{F\beta}$  : Face load factor  
 $K_{F\alpha}$  : Transverse load factor  
 $m_n$  : Normal module  
 $s_{Fn}$  : Normal chordal dimension of root critical section  
 $Y_F$  : Form factor  
 $Y_S$  : Stress correction factor  
 $Y_\beta$  : Helix angle factor  
 $Y_B$  : Rim thickness factor  
 $Y_{DT}$  : Deep tooth factor

#### References

- [1] ISO 6336-3, Calculation of load capacity of spur and helical gears - Part 3: Calculation of tooth bending strength, Corrected version (2007).
- [2] S. M. Vijayakar, H. R. Busby and D. R. Houser, Linearization of multibody frictional contact problems, *Computers & Structures*, 29 (4) (1988) 569-576.
- [3] A. Kahraman and S. Vijayakar, Effect of internal gear flexibility on the quasi-static behavior of a planetary gear set, *Journal of Mechanical Design*, 123 (Sep. 2001) 408-415.
- [4] M. Gulllot and G. V. Tordlon, Stress analysis of thin-rim spur gears by finite element method, *Proceedings of the International Power Transmissions and Gearing Conference*, No. 2 (1989) 26-31.
- [5] D. G. Lewicki and R. Ballarini, Effect of rim thickness on gear crack propagation path, *Journal of Mechanical Design*, 119 (March 1997) 88-95.



**Kyung-Eun Ko** received her B.S. degree in Aerospace and Mechanical Engineering from Korea Aerospace University in 2000 and her M.S. degree in Aerospace Engineering from Seoul National University, Korea, in 2002. Ms. Ko is currently a chief researcher at the Hyundai Heavy Industries, Korea. Her

research interests include machinery design, noise, and vibration.